Wavelet based techniques for textile inspection

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1. INTRODUCTION

Raising quality requirements for manufactured products has led quality control procedures and inspection procedures to an outstanding place in production processes. Industrial sectors related to materials with periodic textured surface (metallic nets, plastic, paper, films, fabric, etc.) are aware of it and are devoting great efforts to this field. Textile fabric is a representative manufactured product of this kind, that presents high quality requirements and challenges for quality control and inspection.

Most defects arising in the production process of a textile material are still detected by human inspection. The work of inspectors is very tedious and time consuming. They have to detect small details that can be located in a wide area that is moving through their visual field. The identification rate is about 70%. In addition, the effectiveness of visual inspection decreases quickly with fatigue.
The technological development has introduced automation in production processes, increasing their productivity and requiring quality control procedures to be automated too. The first automatic on-line quality control devices coupled to weaving machines appeared in the last years. They consist of a set of video cameras and suitable software detecting defects in the fabric. Other off-line devices for fabric defect detection have arisen too. These devices have been tested for simple textile products, mainly solid plane weave. Taking into account that short productions are quite usual and product sequences very diverse, the need is for a robust and versatile defect detection system.

In general, fabric analysis is performed on the basis of digital images of the fabric. Alternatively, there are some works (Ciamberlini et al. [1]) based on the optical Fourier transform directly obtained from the fabric with optical devices and a laser beam.

Digital image processing techniques have been increasingly applied to textured samples analysis over the last ten years. Several authors have considered defect detection on textile materials. Kang et al. [12, 13] analyzed fabric samples from the images obtained from transmission and reflection of light to determine its interlacing pattern. Tsai and Hu [23] used Fourier transforms of solid plane fabric images as the inputs to an artificial neural network for fabric defect detection. They trained the neural network to identify four types of defects: missing pick, missing end, oil fabric stains and broken fabric. In a recent paper, Hu and Tsai [9] have also used wavelet packet bases and an artificial neural network for the stated goals. Wavelets had been previously applied to fabric analysis by Jasper et al. [10, 11]. Escofet et al. [3, 4] have applied Gabor filters (wavelets) to the automatic segmentation of defects on non solid fabric images for a wide variety of interlacing patterns. In the following sections we revise part of this work.

Defects can be classified as local or global. Global defects cause an overall distortion of the basic structure of the fabric and can be detected by means of Fourier analysis. Local defects only affect a small area of the image of the fabric under inspection.

The performance of the Fourier transform based techniques for defect detection is illustrated by two examples, one corresponding to a global defect, the second representing a local defect.
Figure 2 displays digital images of a pattern fabric sample and a defective fabric sample. At first sight, we may consider image (b) affected by a global distortion. Both images are represented by a 256×256 matrix of intensities obtained from a digital camera. Escofet et al. [5] studied the distribution of the peaks in the power spectrum – the squared module of the Fourier transform – of the fabric image and described a procedure to extract the information about its interlacing pattern. Based on this, we expect to perceive some differences between the power spectra corresponding to the samples of Figure 2. The 2D Fourier transforms are evaluated by the FFT algorithm and their modules are displayed in Figure 3 at a logarithmic gray scale.

Millán and Escofet [16] introduced Fourier-domain-based angular correlation as a method to recognize similar periodic patterns, even though the defective fabric sample image appeared rotated and scaled. Recognition was achieved when the maximum correlation value of the scaled and rotated power spectra was similar to the autocorrelation of the power spectrum of the pattern fabric sample. If the method above was applied to the spectra presented in figure 3(a) and 3(b), the maximum angular
correlation value would be considerably lower than the autocorrelation value of the
defect free fabric spectrum. Since the Fourier transform of a periodic structure consists
of delta masses, this decreased correlation can be explained by the blur in the peaks of
the power spectrum of Fig. 3(b) that reveals the presence of a defective fabric. This
effect of blur in the spectrum was also used by Ciamberlini et al. [1] to design optical
filters for defect detection in real time processing.

When dealing with local defects, there are no obvious differences between the power
spectrum of the defective fabric sample and the power spectrum of a defect free fabric.
For example, compare the power spectrum of the pattern fabric sample displayed in Fig.
3(a) with the power spectrum of the defective fabric sample displayed in Fig. 4(b).

For local defects, Tsai and Hu [23] compared the values of the power spectra
corresponding to the peaks – the peaks are the dark points of the power spectra image,
darker points corresponding to higher values in these displays –to discriminate between
defective fabric samples and pattern fabric samples. They trained an artificial neural
network to classify their samples, all of them corresponding to solid plane weave.

Their method to detect local defects in fabrics from the power spectrum images is too
specific and has not been tested with general interlacing patterns other than plane. It
requires a training period for every different weave to be analyzed. Moreover, the
problem becomes more complex when one does not only deal with detection, but also
with location. This means that we would like to make a mark on the defect to find its
position.

![Defective fabric sample](image1.jpg)  ![Power spectrum of the defective fabric sample](image2.jpg)

**Figure 4**
Fourier analysis does not provide, in general, enough information to detect and segment local defects. In our setting, segmentation means to draw a curve enclosing the region containing the defect. Methods that can localize and analyze features in the spatial as well as in the frequency domain, like wavelet methods, are convenient for detecting and segmenting local defects.

2. THE MATHEMATICAL FRAMEWORK IN ONE DIMENSION.

Dennis Gabor [1946] introduced what he called “elementary signals”, now called Gabor functions

$$G_{a,\xi}(t) = g(t-a)e^{2\pi i t \xi}$$

where \( g \) is a gaussian

$$g(t) = \frac{1}{(\pi \sigma^2)^{\frac{1}{4}}} e^{-\frac{t^2}{2\sigma^2}}$$

of energy \( \| g \|_2 = 1 \). If \( f \in L^2(\mathbb{R}) \), one has

$$f(t) = \int_{\mathbb{R}^2} \left< f, G_{a,\xi} \right> G_{a,\xi}(t) da dt.$$

Since \( \left< f, G_{a,\xi} \right> = \left< \hat{f}, \hat{G}_{a,\xi} \right> \), the coefficient can be interpreted as giving joint localization in time and frequency; among other windows \( g \), the gaussian are those providing the maximum joint localization, by Heisenberg inequality.

The above representation is stable, but of course redundant. There are two main inconveniences: first, as it is well known, it is not possible to discretize it and completely eliminate redundancy. Secondly, and more importantly for applications, the analyzing window is fixed, while on an intuitive basis it would be more natural that high frequencies were analyzed with short windows and low frequencies with large ones. Incidentally, another aspect of redundancy is the fact that just the real or imaginary parts of the \( \xi, a \) would suffice as “elementary signals”. It is therefore quite natural to allow the window change its shape, that is, varying \( \sigma \), which is the basic idea in wavelet analysis, where the “elementary signals” are obtained from a fixed function \( \psi \) by translation and scaling:

$$\psi_{a,s}(t) = s^{-\frac{1}{2}} \psi \left( \frac{t-a}{s} \right).$$
In the wavelet setting, a convenient time-frequency analysis is done by **analytic wavelets** $\psi_{a,s}$, corresponding to an analytic $\psi$ (meaning that $\hat{\psi}(\omega) = 0$, for $\omega < 0$) satisfying the **admissibility condition**

$$\int_{0}^{+\infty} \frac{\hat{\psi}(\omega)}{\omega} d\omega = c < +\infty. \quad (1)$$

We set $\psi_s(t) = s^{-1/2} \psi\left(\frac{t}{s}\right)$ and $\varphi_s(t) = \psi_s(-t)^*$, where $z^*$ indicates the complex conjugate of $z$.

For a real function $f \in L^2(\mathbb{R})$, the wavelet coefficients are $Wf(a,s) = \langle f, \psi_{a,s} \rangle = f * \varphi_s(a)$, where $*$ stands for the convolution product. It is by now a classical fact (Mallat [14] 4.3.2) that similarly as before with the $G_{a,s}$ one has the continuous representation

$$f(t) = \frac{2}{c} \text{Re} \left[ \int_{0}^{+\infty} \int_{-\infty}^{+\infty} Wf(a,s) \psi_{a,s}(t) \frac{ds}{s^2} \right] \quad (2)$$

which can also be written

$$f(t) = \frac{2}{c} \text{Re} \left[ \int_{0}^{+\infty} \left( f * \varphi_s * \psi_s \right)(t) \frac{ds}{s^2} \right]. \quad \text{(Calderón formula)} \quad (3)$$

Again, just the real parts $\psi_{a,s}^0 = \text{Re} \psi_{a,s}$ would suffice because one has as well

$$f(t) = \frac{4}{c} \int_{0}^{+\infty} \int_{-\infty}^{+\infty} \langle f, \psi_{a,s}^0 \rangle \psi_{a,s}^0(t) \frac{ds}{s^2}, \quad (4)$$

but expressions like (2) and (3) are more common in electrical engineering and are more suitable to separate the phase and amplitude information.

It is worthwhile pointing out that (1) is equivalent to

$$\int_{0}^{+\infty} \frac{\hat{\psi}(s\omega)}{s} ds = \int_{0}^{+\infty} \frac{(\varphi_s * \psi_s)(\omega)}{s^2} ds = c < +\infty \quad (5)$$

condition that can be interpreted as saying that the “continuous” family of functions $\chi_s = \varphi_s * \psi_s$ covers the whole frequency domain.
A convenient way to look at (3) is considering the piece $D_s(f) = f * \psi_s * \psi_s$ as the detail at scale $s$. Then
\[
P_\tau f(t) = \frac{2}{c} \text{Re} \left[ \int_{\tau}^{+\infty} (f * \psi_s * \psi_s)(t) \frac{ds}{s^2} \right]
\]
represents an approximation of $f$ at scale $\tau$, built from the details at scales $s \geq \tau$. If $\phi$ is the so-called scale function, defined by
\[
\left| \hat{\phi}(\omega) \right|^2 = \int_{-\infty}^{+\infty} \frac{|\hat{\psi}(s\omega)|^2}{s} ds = \int_{-\infty}^{+\infty} \frac{|\hat{\xi}(\xi)|^2}{\xi} d\xi
\]
(complex phase arbitrarily chosen), it is easily seen (Mallat [14], p. 80) that $P_\tau f$ can be written
\[
P_\tau f = \frac{2}{c} \text{Re} \left( \frac{1}{\tau} f * \phi_{\tau} \right)
\]
with, as before, $\phi_{\tau}(t) = \tau^{-1/2} \phi \left( \frac{t}{\tau} \right)$ and $\phi_{\tau}^*(t) = \phi_{\tau}(-t)^*$. Notice that
\[
\left( \frac{1}{\tau} \phi_{\tau} \right)^* \phi_{\tau} = \left( \hat{\psi}(s\omega) \right)^* \hat{\psi}(s\omega) = \int_{-\infty}^{+\infty} \frac{|\hat{\psi}(s\omega)|^2}{s} ds
\]
equals $\int_{-\infty}^{+\infty} \frac{|\hat{\xi}(\xi)|^2}{\xi} d\xi$, so one has the analogue of (5)
\[
\left( \frac{1}{\tau} \phi_{\tau} \right)^* \phi_{\tau} + \int_{0}^{+\infty} (\psi_s * \psi_s) \frac{ds}{s^2} = c
\]
which explains the decomposition, analogue to (3)
\[
f = P_\tau f + \frac{2}{c} \text{Re} \left[ \int_{0}^{+\infty} (f * \psi_s * \psi_s) \frac{ds}{s^2} \right].
\]
If $0 < \tau_1 < \tau_2$, the finer approximation $P_{\tau_2} f$ is obtained from $P_{\tau_1} f$ adding the corresponding details
\[
P_{\tau_2} f = P_{\tau_1} f + \frac{2}{c} \text{Re} \left[ \int_{\tau_1}^{\tau_2} (f * \psi_s * \psi_s) \frac{ds}{s^2} \right]
\]
In terms of (2)
\[
P_\tau f(t) = \frac{2}{c} \text{Re} \left[ \int_{+\infty}^{+\infty} \int_{-\infty}^{+\infty} Wf(a,s) \psi_{a,s}(t) da \frac{ds}{s^2} \right].
\]
Again, \( Wf(a,s) = \langle f, \psi_{a,s} \rangle = \langle \hat{f}, \hat{\psi}_{a,s} \rangle \) gives local information in time and frequency, the joint localization being controlled now by \( \sigma(\psi_{a,s})\sigma(\hat{\psi}_{a,s}) = \sigma(\psi)\sigma(\hat{\psi}) \). To optimize it, it is therefore natural to take as \( \psi \) a Gabor function

\[
\psi(t) = \frac{1}{(2\pi\sigma^2)^{\frac{1}{4}}} e^{\frac{t^2}{2\sigma^2}} e^{2\pi i \eta t}.
\]

If \( \eta\sigma >> 1 \), \( \psi(\omega) = (4\pi\sigma^2)^{1/2} e^{-2\pi^2\sigma^2(\omega-\eta)^2} \) is very small for \( \omega < 0 \), whence \( \psi \) can be considered as an approximate analytic wavelet, called a Gabor wavelet.

These are the wavelets we will be using. Besides the optimality of the joint localization, another important reason for this choice is that 2D Gabor wavelets seem suitable to modelize the receptive fields of the simple cells of the human visual cortex, as proposed by Marcelja [15] and Daugman [2]. Gabor wavelets have been extensively used in a wide range of problems in image processing: image coding and compression, image enhancement and reconstruction, denoising, etc. (see for instance Navarro, Tabernero and Cristobal [18]). Other authors have proposed differences or derivatives of gaussians (mexican hats) as alternatives.

Before describing how this is used in applications we briefly address the question of discretization, still at a theoretical level.

All representations above, although stable, are of course highly redundant. It is well known, in contrast with the situation in the windowed Fourier transform, that for some mother wavelets \( \psi \) it is possible to discretize the integrals in (2), (9) and completely eliminate the redundancy. These are the so-called \textit{orthonormal wavelets} for which the discretization \( s = 2^{-j}, \ a = n2^{-j}, \ n \in \mathbb{Z} \) (which is heuristically justified by Shannon’s sampling theorem, see next section) leads to a genuine orthonormal basis of \( L^2(\mathbb{R}) \). The resulting representation has the inconvenience, though, of not being translation invariant.

Unfortunately, most wavelets, and specifically the Gabor wavelets, are not orthonormal. In such case a discretization is done only in the scale, say \( s = 2^{-j} \) again; this leads to a
translation-invariant redundant representation (dyadic wavelet representation). For
this representation to hold, the following discrete analogue of (1) must be fulfilled
\[ 0 < c_1 \leq \sum_{j=-\infty}^{\infty} |\hat{\psi}(2^{-j}\omega)|^2 \leq c_2 < +\infty, \]
that is, the dyadic wavelets \( \psi_j(t) = 2^{-j/2} \psi(2^{-j}t) \) cover the whole frequency domain.
Indeed it is well-known Mallat S. [14] that one has then a stable representation of \( f \), of the type
\[ f(t) = \sum_{j=-\infty}^{\infty} 2^j \left( f * \overline{\psi_j} \right)(t) \]
with a convenient dual wavelet \( \overline{\psi}_j \) (possibly \( \psi_j \neq \psi_j \)).
The approximation of \( f \) at a scale \( s = 2^{-k} \) is then
\[ P_k f(t) = \sum_{j:k} 2^j \left( f * \overline{\psi}_j \right)(t) \]
In case of an orthonormal wavelet basis \( \psi_{n,j} = \psi_{n2^{-j},2^{-j}} \),
\[ P_k f(t) = \sum_{j:k} \sum_n \langle f, \psi_{n,j} \rangle \psi_{n,j}(t). \]
A third possible discretization is through frame-theory. A condition slightly stronger
than (1) implies that the \( \psi_{n,j} \) form a frame, meaning that for every \( f \in L^2(\mathbb{R}) \),
\[ c_1 \sum_j \sum_n |\langle f, \psi_{n,j} \rangle|^2 \leq \int |f(t)|^2 \, dt \leq c_2 \sum_j \sum_n |\langle f, \psi_{n,j} \rangle|^2. \]
In this case there exist “dual functions” \( \overline{\psi}_{n,j} \) such that
\[ f(t) = \sum_j \sum_n \langle f, \psi_{n,j} \rangle \overline{\psi}_{n,j}(t). \]
And the approximations are, as before, \( P_k f(t) = \sum_{j:k} \sum_n \langle f, \psi_{n,j} \rangle \overline{\psi}_{n,j}(t) \).

3. THE MODIFIED MATHEMATICAL FRAMEWORK FOR THE APPLICATIONS.

In the applications, exact reproduction formulas valid for general functions are in fact not needed. The context for applications can be described as follows: the information at hand, the function one wants to process, is interpreted as the approximation at a certain scale, say 1, of a given function, \( P_1 f \). In practice, this starting information is finite-dimensional, because it comes through some finite number of samples of \( f \). Thus, \( P_1 f \)
has components only in low frequencies, and belongs to a space of finite dimension. This space depends on the building blocks or elementary signals that are being used; for instance, in the context of Fourier analysis, where the building blocks are complex exponential functions, this space would be that of trigonometric polynomials of a certain degree.

A typical aim in the applications is to fix a dyadic scale \( s = 2^k, k > 0 \), and find the coarser approximation \( P_kf \) and the dyadic details \( f * \psi_j * \psi_j \) for the intermediate scales \( 2^j, j=1,...,k \), discretizing (8). In case of orthonormal wavelets or frames, the theory says that these details are completely determined in a stable way, at all levels, by their values on \( n2^j, n \in \mathbb{Z} \), and are efficiently computed with Mallat’s algorithm.

The following remarks apply:

- There is no need that the scale function \( \phi_k \) matches the one above. Any \( \phi_k \) satisfying
  \[
  0 < c_1 \leq |\hat{\phi}_k(\omega)|^2 + \sum_{j=1}^{k} |\psi(2^j \omega)|^2 \leq c_2 < +\infty
  \]
  in the region where the frequency content of \( f_1P \) lies would do. Put in a different way, we are decomposing \( P_t f \).

- In order to make the above easier to fulfil, one may take not one but a finite family \( \psi^{(i)},...,\psi^{(i)} \) of wavelets such that
  \[
  0 < c_1 \leq |\hat{\phi}_k(\omega)|^2 + \sum_{j=1}^{k} \sum_{i=1}^{L} |\psi^{(i)}(2^j \omega)|^2 \leq c_2 < +\infty
  \]
  so that one would have a theoretical reconstruction (we are avoiding real parts)
  \[
  P_t f = f * \phi_k * \psi_k + \sum_{j=1}^{k} \sum_{i=1}^{L} 2^j \left( f * \psi^{(i)}_j * \psi^{(i)}_j \right). \tag{13}
  \]
  This point will be important later when we add wavelets according to different orientations.
• The orthonormal or frame condition is replaced by the following heuristic argument: the wavelets \( \psi^{(i)} \) being well localized in frequency, say in \( 0 < \omega < 1 \), \( \hat{\psi}^{(i)}_j \) is supported in \( 0 < \omega < 2^{-j} \) and \( f \star \psi^{(i)}_j \) as well, whence by Shannon’s theorem, it is enough to sample it at \( n2^j, n \in \mathbb{Z} \). That is, we will only sample \( f \star \psi^{(i)}_j(n2^j) \) which corresponds to the wavelet coefficient at \( s = 2^j, a = n2^j \).

In practice, one wants to analyze finite signals of size \( N = 2^L \), \( a_0[n], n=0, \ldots, N-1 \), which are interpreted as the sampled values at the integers of an \( N \)-periodic function. As said before, we consider that these samples give us a 1-scale approximation of \( f, \ P_1f \). This might be a trigonometric polynomial (computed with the DFT) or something similar in order to avoid border distortion. We may assume that \( P_1f \) has frequency content in \([–\_, \_\,] \) say.

All representation-decompositions above split periodic functions into periodic functions when computing a convolution

\[
f \star \psi_j(x) = \int_{-\infty}^{+\infty} f(t) \psi_j(x-t) \, dt
\]

strictly speaking each \( \psi_j \) should be \( N \)-periodized.

\[
f \star \psi_j(x) = \int_{1}^{N} f(t) \psi_j^{per}(x-t) \, dt, \quad \psi_j^{per}(t) = \sum_{k=-\infty}^{+\infty} \psi_j(t+kN)
\]

In our concrete situation this will be neglected, though.

When computing the wavelet coefficient \( Wf(n2^j, 2^j) \),

\[
f \star \psi_j(2^j) = \int_{0}^{N} f(t) 2^{-j/2} \psi(2^{-j}t-n) \, dt = \int_{0}^{2^jN} 2^{j/2} f(2^j t) \psi(t-n) \, dt
\]

this is of course approximated by \( \sum_{m=0}^{N} f(2^j m) \psi(m-n) \), that is, we apply the same discrete filter \( \psi[m-n] \) to decimated versions of the original discrete signal. This amounts to a typical pyramid algorithm, as Mallat’s one.
A final observation is concerning the reconstruction formulas like (13). Consider a detail \( D_j(f) = f * \psi_j * \psi_j \) at scale \( 2^j \) at a point \( t \),

\[
D_j(f)(t) = f * \psi_j * \psi_j(t) = \int Wf(2^j, x) \psi_j(t - x) \, dx
\]

or an approximation at scale \( 2^k \)

\[
P_k(f)(t) = f * \phi_k * \phi_j(t) = \int \left( f * \phi_k \right)(x) \psi_j(t - x) \, dx.
\]

In practice these integrals are not evaluated. They are approximated by the single contribution at \( x = t \), that is, in the application there is an identification between the details \( D_j(f)(t) \) and the wavelet coefficient \( Wf(2^j, t) \), and the same fact applies to the approximations. This procedure is justified by the stability of the representation.

4. OUR GABOR WAVELETS IN TWO DIMENSIONS.

The framework described in the previous sections, or rather its analogue in dimension two, with Gabor wavelets, is the one used by Navarro and his group in several applications in image processing [18].

![Gabor wavelet](a) Gabor wavelet (real part)

![Fourier transform](b) and its Fourier transform

Figure 5

The idea that a fabric can be considered as a textured material lead us to the application of their techniques to the detection and segmentation of defects. Specifically, the problem of detecting local defects in a surface can be related to texture segmentation ([20], [22], [24] and [25]) when either the material, the defect or both are textured, as in textile webs.
The general form of a Gabor wavelet in two dimensions is

$$\psi(x) = \frac{1}{\pi \sigma^2} e^{-\frac{1}{2} \frac{|x|^2}{\sigma^2}} e^{2\pi i \eta \cdot x},$$

where $x = (x_1, x_2)$ and $\eta = (\eta_1, \eta_2)$. Its Fourier transform is

$$\psi(\omega) = \left(\frac{4\pi \sigma^2}{\omega_0}\right)^{\frac{1}{2}} e^{-2\pi \sqrt{\sigma^2} \cdot |\omega - \eta|^2}.$$

Next figure shows the real part of a Gabor wavelet and its corresponding Fourier transform at two scales. In the spatial domain (above), black corresponds to negative values, white to positive values and gray to zero. In the frequency domain (below) white corresponds to positive values and black to zero.

![Image of Gabor wavelet and its Fourier transform](image-url)

Figure 6. Scaling effect on the Gabor function and its corresponding Fourier transform

We will take finitely many $\psi^{(i)}$, ..., $\psi^{(l)}$ corresponding to $\eta^{(i)}$, ..., $\eta^{(l)}$ and a $\phi_k$ such that (12) holds in some region $|\omega| \leq c$. The choice of the $\eta^{(i)}$ is motivated by experimental results about the visual system. Heuristically, we want the decomposition to be done in
the range of frequencies that the visual system is able to discriminate; for discrete images, the higher frequency is considered to be 1 cycle/2 pixels. Taking into account that the human visual system performs well in object localization but no so in frequency analysis, Navarro et al. choose half this frequency. That’s why we set \( |\eta^{(i)}| = \frac{1}{4} \). Lower frequencies will be covered by scaled versions of the wavelet (Fig. 7(a)).

To modelize the response of the human visual system to different orientations, the points \( \eta^{(i)} \) are distributed angularly. We chose \( l=4 \), corresponding to the angles \( \theta = 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4} \). Recall that the analysis will be done with the real parts of these wavelets and that those have an even Fourier transform (Fig. 5(b)), so the eight principal directions are covered (Fig. 7(b)).

Our next parameter is \( k \), the number of layers. The usual in visual applications is \( k=4 \). Next the parameter \( \sigma \) must be chosen so that (12) is fulfilled in \( |\omega| \leq \frac{1}{4} \), except in some neighborhood of 0, that will be occupied by our scaling function \( \phi_k \). A convenient choice of \( \sigma \) turns out to be \( \sigma = 2.25 \). Figure 7(b) displays the covering of the frequency domain obtained from scaling and rotating the real part of such a Gabor wavelet. The scaling function \( \phi_k \) is chosen as another gaussian with no modulation, concretely
It is a cubic B-spline low-pass filter, that will be used in the segmentation algorithm. Its Fourier transform covers the black hole in the center of figure 7(b).

In one dimension, the computation of the wavelet coefficients involved only one filter that was applied to decimated versions (by factors of \(2^j\)) of the original \(N\)-sample. In 2D, we start from an image of size \(N^2\). Since the wavelets \(\psi^{(1)}, \psi^{(2)}, \psi^{(3)}, \psi^{(4)}\) are all rotated one from each other, again all computation depend only on rotated versions of one filter \(\psi[m,n]\), this time being applied to decimated versions of the original image. That filter turns out to be the one corresponding to \(\eta^{(1)} = (1/4, 0)\) (real part), and neglecting small values after convenient multiplication by a constant term:

\[
\phi_{k} = \begin{bmatrix}
1 & 4 & 6 & 4 & 1 \\
4 & 16 & 24 & 16 & 4 \\
6 & 24 & 36 & 24 & 6 \\
4 & 16 & 24 & 16 & 4 \\
1 & 4 & 6 & 4 & 1
\end{bmatrix}.
\]

(14)

The rotated filters will be called \(\psi^{(2)}, \psi^{(3)}, \psi^{(4)}\). Notice that the even columns are zero (because of \(\cos(2\pi \frac{1}{4}\, x)\)) and that \(\psi\) is a center symmetric matrix. Notice too that \(\sum_{m,n} \psi_{m,n} \approx 0\), corresponding to \(\int\int \psi \, dx \, dy \approx 0\), and the rows add up to 0 approximately.

This properties are related to the performance of the filter.

Recall that approximate reconstruction formulas can be obtained with either the real part or the imaginary part of a Gabor wavelet. The imaginary part giving rise to a different
set of filters $\gamma^{(1)}, \gamma^{(2)}, \gamma^{(3)}, \gamma^{(4)}$. So, the details obtained will be different in each case. The details corresponding to the real part enhancing some features of the image different from those enhanced by the imaginary part. So it may be a matter of consideration which one is going to be used in every application. In the segmentation procedure described below, we use both filters to take advantage of the specific feature extraction corresponding to each one.

5. SEGMENTATION

This section describes the procedure we propose for defect detection in regular textures. There is an extensive literature suggesting that Gabor wavelets are good descriptors of texture: Escofet [3,4], Navarro et al. [18], Turner [24], Fogel and Sagi [6], Weldon et al. [25], Nestares et al. [19], Portilla et al. [20]. A visible defect will cause a local change of visual texture. If the proposed Gabor wavelets are able to accurately describe texture, then we can exploit this fact to automatically segment the image and detect visually relevant defects. The presented method must meet three requirements. Firstly, we have to enhance changes in the descriptors, which may correspond to a defect in such a way that a binarization makes possible the segmentation of defective areas from the textured background. Secondly, the process must integrate defects captured in different orientations and scales of the Gabor filters into a single binary map as the output with the location of defects. Thirdly, the procedure must be automatic, robust and versatile, easily adaptable to a variety of regular textures of different materials. This third aspect entails that we will not introduce key-parameters, which may require specific adjustments of parameters or procedures to a particular kind of defect or texture. In this way, we will avoid potential problems of overtraining or undertraining which frequently appear when optimizing a given method with a limited set of training samples.

Figure 8(a) shows a schematic diagram of the procedure. It starts with an image of the sample to inspect $t(x, y)$. To combine the feature extraction of the real part of the Gabor wavelet and the imaginary part of the Gabor wavelet, we are going to combine the details corresponding to each one. This combination is achieved applying complex filters to the original image. These complex filters are those corresponding to the complex Gabor wavelet. Their real parts are $\psi^{(1)}, \psi^{(2)}, \psi^{(3)}, \psi^{(4)}$ and their imaginary
parts $\gamma(1), \gamma(2), \gamma(3), \gamma(4)$. These complex filters are applied to the original image and to decimated versions of the original image (including a convolution with the low-pass filter cubic B-spline before each subsampling to avoid aliasing, a common practice in digital image processing). If we let $p=0,1,2,3$ represent scaling by $2^p$ and $q=0,1,2,3$ represent the four rotations previously indicated, $t_{pq}(x, y) = D_p^q(t)(x, y)$, are the complex details at scale $2^p$ and orientation $q \pi / 4$. The coarser approximation obtained after the fourth decimation is the low pass residual image $t_{LPR}(x, y)$.

(a) main diagram
In the second step, our texture descriptors are obtained by expressing the module of the complex details \( |t_{pq}(x, y)| \), that we call the filtered image, in contrast units. This is accomplished by dividing every \( |t_{pq}(x, y)| \) by the resized low-pass residual image \( t_{LPR} \).

Thus, the set \( \{p, q\} \) of features \( T_{pq}(x, y) \) are given for each pixel \( (x, y) \) by the expression

\[
T_{pq}(x, y) = \frac{|t_{pq}(x, y)|}{t_{LPR}(x', y')} ,
\]

with (see Fig. 8)

\[
x' = 1 + I \left( \frac{x - l}{2^p} \right) , \quad y' = 1 + I \left( \frac{y - l}{2^p} \right) ,
\]

where function \( I(z) \) means the integer part of argument \( z \).

Before analyzing the texture to be tested, we first apply the same procedure to a prototype defect-free sample \( r(x, y) \) and store the mean and standard deviation of the
histograms of each feature (entry on the left in Figure 8(a) and sketched in detail in Figure 8(b)). We assume that both, the image of the texture under inspection \( t(x, y) \) and the image of the reference non defective texture \( r(x, y) \), are acquired under the same experimental conditions of scale, orientation and resolution. In Figure 8(b), the filtered images in absolute value \( |r_{pq}(x, y)| \) are again converted to contrast units, by dividing by the corresponding LPR image \( r_{LPR}(x', y') \)

\[
R_{pq}(x, y) = \frac{|r_{pq}(x, y)|}{r_{LPR}(x', y')} ,
\]

in the same way as in Eq. (15). The mean value (over all the pixels) of each \( R_{pq} \) and the standard deviation \( \sigma_{pq} \) are calculated by standard expressions

\[
\bar{R}_{pq} = \frac{1}{N_p^2} \sum_{x=1}^{N_p} \sum_{y=1}^{N_p} R_{pq}(x, y),
\]

\[
\sigma_{pq} = \sqrt{\frac{1}{N_p^2} \sum_{x=1}^{N_p} \sum_{y=1}^{N_p} (R_{pq}(x, y) - \bar{R}_{pq})^2},
\]

where \( N_p^2 \) is the number of pixels of the filtered image in the resolution level \( p \). The two sets of sixteen (4x4) values \( \{\bar{R}_{pq}\} \) and \( \{\sigma_{pq}\} \) are the reference entry to the main procedure on the left of the scheme in Figure 8(a).

The next step is to compare, for each pixel or location, the features of the sample under study with those of the reference. The closer the values, the higher the likelihood of it coinciding with the prototype, and conversely, the larger the difference the higher the probability of there being a defect. Thus, we calculate, for each level \( p \) and orientation \( q \), the magnitude of the difference between features of the sample under analysis and the mean of the prototype

\[
d_{pq}(x, y) = |r_{pq}(x, y) - \bar{R}_{pq}|\]
In order to reduce noise, for each pixel we set as zero those differences $d_{pq}(x, y)$ below a threshold; i.e. for those values with a high likelihood of being like the prototype. We consider a standard thresholding operation given by the expression:

$$
S_{pq}(x, y) = \begin{cases} 
    d_{pq}(x, y), & \text{if } d_{pq}(x, y) \geq \tau \sigma_{pq}, \\
    0, & \text{otherwise}
\end{cases},
$$

where the threshold is proportional to the standard deviation $\sigma_{pq}$ calculated from the reference feature array $R_{pq}(x, y)$. We take a fairly standard constant value $\tau = 3$ according to a low risk criterion: only points with differences above three times the standard deviation are eligible as defects, which strongly reduces the probability of misclassifying points of the background (regular texture) as defective areas. The resulting array of the thresholded feature differences is represented by $S_{pq}(x, y)$ in the diagram in Figure 8(a).

For each scale level $p$ and for every pixel $(x, y)$, a vector of four components $S_{p}^{xy} = \{(S_{p}^{xy})_q\}$ with $q = 0…3$ can be built. Each component of the vector $S_{p}^{xy}$ is defined by $(S_{p}^{xy})_q \equiv S_{pq}(x, y)$ and coincides with the thresholded feature difference of pixel $(x, y)$ in the scale level $p$ and orientation $q$. In the next stage an array $K_p(x, y)$ is calculated for each scale level $p$ with the norm of vectors $S_{p}^{xy}$, that is,

$$
K_p(x, y) = \left| S_p^{xy} \right| = \left\{ \sum_{q=0}^{3} \left[ S_{pq}(x, y) \right] \right\}^{1/2}.
$$

The definition of $K_p$, i.e., the norm of the feature difference vector is a common metric used in standard clustering algorithms for segmentation. According to Eq. (22), the array $K_p(x, y)$ concentrates the information on the likely defective areas obtained in the four orientations $q = 0…3$ in a single array for the scale level $p$. Thus, the result of this stage is a set of four images $K_p(x, y)$ with $p = 0…3$.

In the next two stages we combine the information coming from the four different resolution levels $p$. To this end the resized version of each $K_p(x, y)$ array is prepared. In
order to avoid false alarms we consider that a defect must appear in at least two adjacent resolution levels. As a simple way to implement a logic “and”, assuming that $K_p(x,y)$ is proportional to the probability of there being a defect, we then calculate the geometric means of every pair of adjacent levels given by the formulas:

\[
K_{01}(x,y) = \left[ K_0(x,y)K_1(x,y) \right]^{1/2},
\]
\[
K_{12}(x,y) = \left[ K_1(x,y)K_2(x,y) \right]^{1/2},
\]
\[
K_{23}(x,y) = \left[ K_2(x,y)K_3(x,y) \right]^{1/2}.
\]

This operation reduces false alarms yet preserves most of the defective areas. Now we combine the resulting $K_{01}(x,y)$, $K_{12}(x,y)$ and $K_{23}(x,y)$ in a logic “or”, simply as the arithmetic mean, to account for defects detected at different scales:

\[
K(x,y) = \frac{1}{3} \left( K_{01}(x,y) + K_{12}(x,y) + K_{23}(x,y) \right)
\]

The array $K(x,y)$ contains the joint contribution of the sixteen $pq$-channels.

The last stage corresponds to the binarization of $K(x,y)$ to provide an image $B(x,y)$ where local defects (objects) appear segmented from the regular texture (background). This is achieved by thresholding $K(x,y)$. Values below the threshold are considered as belonging to the background and values above the threshold are considered as belonging to defective areas.

This threshold value is not critical and can be estimated in different ways. One possible way is to calibrate the system at the beginning of the process by applying the procedure to an additional piece of faultless texture whose image would be the input image $t_0(x,y)$. In this case the obtained array $K_0(x,y)$ should contain very low values. An estimation of the threshold $U$ as $U = \bar{K}_o + \rho \sigma_o$ with $\bar{K}_o$ being the mean value of $K_0(x,y)$, $\sigma_o$ its standard deviation and $\rho$ a standard constant of value $\rho = 3$, provides an appropriate threshold value for binarization. Alternatively, a simpler way is to calculate
\[ U = \left( \frac{\rho}{16} \right) \sum_p \sum_q \sigma_{pq}, \] which is proportional to the mean value of the sixteen standard deviations \( \sigma_{pq} \) with a constant of proportionality equal to a standard value, for example, \( \rho = 3 \). An opening operation with a small mask of 3x3 pixels helps to remove the remaining isolated noisy points from the binary output image \( B(x, y) \).

6. RESULTS

We have applied this segmentation procedure to a variety of textile webs with different structures (plain, twill, etc.) and with yarns of different colors affected by common local defects. These defects are caused by missing or broken yarns or by changes in tension during production in the loom. The defects display a variety of shapes: line, spot, band, ladder, hole, etc. In this section we show the results with representative examples chosen from among those mentioned.

Before applying the algorithm it is important to fix the acquisition conditions, not only in terms of uniformity but also of scale and resolution. We consider the maximum frequency \( f_{\text{max}} = f_4 = 1/4 \text{ yarn/pixel} \). This means that a woven yarn is digitized into four pixels on average. If the yarns in the weft and warp directions are of different thickness, the camera is adjusted to fit the thinnest yarn to four pixels. In our experiments this adjustment was made manually. Adjustment of lightness and scale is reasonably easy and only needs to be done once unless we change the web. The images of the textile samples we analyzed are 256x256 pixel size.

Figure 10(a) shows a sample of twill fabric containing some defects. The yarns in the warp are of a different color from the yarns in the weft. The defect appears as aligned spots, although some isolated spots can also be found. Our algorithm for defect detection is applied to the image in Figure 10(a), which is taken as the entry \( t(x, y) \). Figures 10(b)-(e) show the decompressed versions of the arrays \( K_p(x, y) \), for the resolution levels \( p = 0...3 \). Figure 10(f) is the image \( K(x, y) \) with the joint contribution of all the pq-channels and Figure 10(g) is the binary image \( B(x, y) \), which is the thresholded binary version of \( K(x, y) \) and constitutes the output image. It can be seen that both the aligned and the isolated defective spots are correctly segmented from the background in Figure 10(g).
Figure 10 (a) Defective twill fabric (multiple threads broken); (b) to (e) decompressed versions of the arrays $K_p(x,y)$ for the resolution levels $p = 0,...,3$ respectively; (f) array $K(x,y)$ with the joint contribution of the $pq$ channels; (g) binary output image $B(x,y)$.  

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Figure 11 (a) Thin-place effect in a twill fabric; (b) to (e) decompressed versions of the arrays $K_p(x,y)$ for the resolution levels $p = 0,\ldots,3$ respectively; (f) array $K(x,y)$ with the joint contribution of the $pq$ channels; (g) binary output image $B(x,y)$. 

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An interesting case is shown in Figure 11. A sample of twill fabric contains a defective band in the central part of the image (Fig. 11(a)). The defect is called thin-place, and is caused by a lower density of filling yarns in this band. Figures 11(b)-(e) are again the decompressed $K_p(x,y)$ with $p = 0...3$. Figure 11(f) is the array $K(x,y)$ with the joint contribution of channels, and Figure 11(g) is the final binarized image $B(x,y)$. Although the defective band is clearly segmented in the final result, in this case the particular resolution level $p = 1$ (Fig. 11(c)) alone provides a better intermediate result. In this particular example, the channel $p = 1$ clearly provides the best tuning of the defect out of the four resolution channels. The later operations (multiplication and addition), designed for the sake of automatism and robustness of method, to reduce noise and to integrate information from the four resolution channels, have the drawback of mixing channels that are very well tuned with the defect with others having no information. As a result, the quality of the segmentation is not so good as it could be if we chose the best channel alone. However, with this mixing procedure we gain robustness. The benefits of high robustness and automatism, regardless of the type of web or defect are much more important than a perfect segmentation.

In the following figures we present the input image of a fabric to inspect $t(x,y)$ (a) together with an image of the joint contribution channels $K(x,y)$ (b) and the final output image $B(x,y)$ (c). Figures 12 and 13 correspond to twill fabrics with defects along a line: a missing yarn (mispick) and a double yarn, respectively. In both cases the output images contain the defects correctly discriminated from the background. Figures 14 and 15 correspond to twill samples with defects in a dotted distribution: several broken yarns and a down heddle defect respectively. The broken yarns are correctly segmented in Figure 14(b). The small size of the defects in the down heddle defect makes for difficulty in detecting some dots and discriminating them from the background. Careful observation of the array $K(x,y)$ with the joint contribution of channels in Figure 15(b) allows us to locate all the defects. The result of the final opening operation is, in this case, that some points are removed (Fig. 15(c)). However, more than 50% of point defects (8 out of 15) are detected by applying the general method.

The plain fabric in Figure 16(a) has two spots of very different intensity. The spots are quite big in comparison with previous dotted defects. Both spots are successfully segmented in Figures 16(b) and 16(c).
Finally, Figure 17 contains a fabric with yarns of the same color. The defect is due to the crossed breaking of some yarns in both the warp and weft directions. After processing, the defect in the two perpendicular directions is correctly segmented (Figs. 17(b) and 17(c)).

**Figure 12**
(a) Twill fabric with missing yarn (mispick);
(b) $K(x,y)$; (c) output image $B(x,y)$.

**Figure 13**
(a) Twill fabric with double yarn defect;
(b) $K(x,y)$; (c) output image $B(x,y)$. 
Figure 14
(a) Twill fabric with broken yarns; (b) $K(x,y)$; (c) output image $B(x,y)$.

Figure 15
(a) Twill fabric with down heddle defect; (b) $K(x,y)$; (c) output image $B(x,y)$.
Figure 16
(a) Large defects of different colors in a plain fabric with black and white threads;
(b) \( K(x,y) \); (c) output image \( B(x,y) \).

Figure 17
(a) White twill fabric with crossed break of multiple threads;
(b) \( K(x,y) \); (c) output image \( B(x,y) \).
7. CONCLUSIONS.

The method proposed for local defect detection has been shown to be a useful tool for inspecting industrial materials with periodic regular texture. The method is based on a multiscale and multiorientation Gabor filter scheme that roughly imitates the early human vision process.

As we intended, a general improvement and enlargement of the vision system capabilities can be achieved by using the proposed algorithm to detect local defects in regular textures. Versatility, full automatism, computational efficiency, robustness and industrial applicability were the pursued properties of the method and we have demonstrated them through a selection of results obtained from textile inspection.

We have built an algorithm for the automatic application of the method to an input image of the sample under inspection. The algorithm applies the Gabor filter scheme in the spatial domain following a fast pyramid implementation for computational efficiency. An image with the joint contribution of the complete set of multiresolution and multiorientation channels is binarized. In the binary output image local defects appear segmented from the background.

One of the most important advantages of the method is that it is multipurpose without requiring any adjustment. The only considerations that require attention are optical conditions such as lightness and scale to guarantee optimal performance, and a preliminary analysis of a prototype defect-free sample to extract the mean and standard deviation of its texture descriptors. We have avoided the use of adjustable weighting functions or parameters that might make the inspection process too dependent on adjustment to a particular reduced set of textures or defects. The method is robust. It is resistant against common input variations such as changes of illumination. It works with contrast rather than luminance units and therefore it should work well under reasonable changes of brightness level. Furthermore, it can be applied to composite patterns with elements of different brightness without any particular adaptation. In addition, there is not any preferable orientation in which the texture has to be fixed before applying the method.
We have tested the proposed method to a wide variety of defective fabric samples obtaining, in general, very good results. We have presented several representative cases where different shapes, structures, colors, sizes, etc. of defects and textured background have been correctly segmented.

The versatility of the method has been demonstrated not only by its applicability to different regular textures but also, for a given texture, the method allows to detect a variety of defects. The method does not need human supervision nor previous knowledge about the texture or defect. In fabric inspection, for example, it does not need information of the repeat pattern in comparison with the method proposed by Jasper et al. [11].

The results of defect detection in fabrics shown and discussed in this paper lead - as first application - to textile inspection. Except for minor adaptations to each particular case, the method is ready to be used in an on-line industrial inspection system.

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9. REFERENCES.


